# Resynthesis of Coupled Piano Strings Vibrations Based on Physical Modelling

M. Aramaki, J. Bensa, L. Daudet, Ph. Guillemain, R. Kronland-Martinet
CNRS-Laboratoire de Mécanique et d'Acoustique
31 ch. Joseph Aiguier, 13402 Marseille Cedex 20, France
{aramaki, bensa, daudet, guillem, kronland}@lma.cnrs-mrs.fr

**Abstract**

This paper presents a technique to resynthesize the sound generated by the vibrations of two piano strings tuned to a very close pitch and coupled at the bridge level. Such a mechanical system produces doublets of components generating beats and double decays on the amplitudes of the partials of the sound. We design a waveguide model by coupling two elementary waveguide models. This model is able to reproduce perceptually relevant sounds. The parameters of the model are estimated from the analysis of real signals collected directly on the strings by laser velocimetry. Sound transformations can be achieved by modifying relevant parameters and simulate physical situations.

## 1. Introduction

Synthesis of piano tones by physical modelling requires the simulation of an intricate chain which begins by the finger of the pianist, the hammer–string impact, the propagation of string vibrations, the interaction between strings and bridge and the acoustic field radiation by the soundboard [1]. In this study, we focus on the resynthesis of the vibration generated by a system of two coupled piano strings using waveguides synthesis models [2]. Indeed, in a real piano, two strings (or three for medium and high pitch) tuned to a very close pitch are used to produce the same note. Moreover, each string is tightened between a fixed support (the nut) and a mobile support (the bridge). Thus, the bridge permits interactions between strings and this coupling generate audible phenomena like beats [3]. They constitute one of the most important features of the piano sound from a perceptive point of view. Previous work [4] has shown that by coupling two elementary waveguides, the vibrations of a single string with two polarisations (horizontal and vertical) could be accurately reproduced. This coupling phenomenon between orthogonal polarisations for one string is similar to the interaction between two strings belonging to the same doublet and coupled through the bridge. In this article, we shall first describe this coupled waveguide model and see how it can be adapted to the case of two strings. For that purpose, we shall consider only the vertical polarisations which represent the most important contribution to the resulting sound. The parameters of this model are estimated from the analysis of real signals. We shall end up by discussing how these parameters can be changed to simulate various physical configurations.

## 2. Waveguide model for two coupled strings

We consider a system of two strings belonging to a same doublet and coupled through the bridge. The synthesis model used to simulate the behaviour of this system is similar to the model used for a string with two polarisations, which is designed by coupling two elementary waveguide models represented in figure 1.

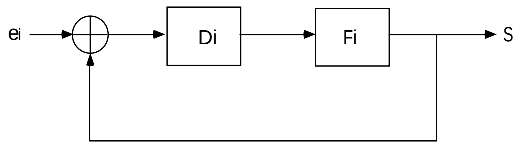


*Fig 1 : Elementary waveguide model representing a single string in one polarisation. $e_i$ is the input of the model and s the output. i indicates the number of the string (1 or 2). The filter $D_i$ represents the propagation delay for the vibration to go back and forth along the string. The filter $F_i$ represents all the dissipation and dispersion phenomena.*

The transfer function of one elementary waveguide model is :

$$G_i = \frac{s_i(\omega)}{e(\omega)} = \frac{F_i(\omega)e^{-i\omega D_i}}{1 - F_i(\omega)e^{-i\omega D_i}}$$

We now couple two such waveguides by feeding the input of one by the filtered output of the other one (figure 2).

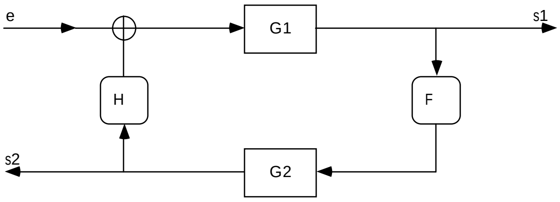


*Fig 2 : Coupled waveguide model. e represents the input of the model, $s_1$ is the output at the string 1 level (the excited string) and $s_2$ the output at the string 2 level (the string excited by the coupling). Each element $G_1$ and $G_2$ represents one string (indexed by 1 and 2) and is an elementary waveguide model as presented on figure 1. H and F are the coupling filters.*

The coupling elements of the model are the filters F and H. We assume that this coupling is localised at one end of the strings, namely the bridge. The filters F and H are complex-valued. This type of model has already been proposed. In particular, Tolonen and al. models [5] take into account multiple strings in the guitar case. These models can reproduce some of the effects generated by the interaction between strings. Nevertheless, as the coupling elements of the model are constant real gains, they give the same behaviour for all partials. In our model, F is related to the amount of energy transferred from string 1 to string 2, and similarly H is related to the amount of energy transferred from string 2 to string 1. As we are interested in describing the coupling, we assume that only one string is excited. The transfer functions of the coupled waveguide model are given by:

$$T_1(\omega) = \frac{s_1(\omega)}{e(\omega)} = \frac{G_1(\omega)}{1 - H(\omega)F(\omega)G_1(\omega)G_2(\omega)} \text{ and}$$
$$T_2(\omega) = \frac{s_2(\omega)}{e(\omega)} = \frac{F(\omega)G_1(\omega)G_2(\omega)}{1 - H(\omega)F(\omega)G_1(\omega)G_2(\omega)}$$

We notice that the two transfer functions $T_1(\omega)$ and $T_2(\omega)$ have the same denominators and one can show that they generate the same resonance frequencies. In the time domain, this model generates two signals, which are sums of exponentially damped sinusoids:

$$T_1(t) = H(t)\sum_{k=-\infty}^{k=+\infty} a_k e^{-\alpha_k t} e^{i\mu_{1k}t} + b_k e^{-\beta_k t} e^{i\mu_{2k}t} \text{ and}$$
$$T_2(t) = H(t)\sum_{k=-\infty}^{k=+\infty} c_k e^{-\alpha_k t} e^{i\mu_{1k}t} + d_k e^{-\beta_k t} e^{i\mu_{2k}t}$$

where H(t) is the Heaviside function. The parameters $a_k$, $b_k$, $c_k$, and $d_k$ are the amplitudes of each component. The terms $\alpha_k$ and $\beta_k$ are the two damping coefficients. $\mu_{1k}$ and $\mu_{2k}$ represent the resonance frequencies of mode k. All these parameters can be expressed as functions of the coupled waveguide parameters. By putting $D_1=D_2$ (which corresponds to two strings with the same length), one can show that the partials are grouped into doublets of components. The frequency gap is typically in the range [0.1Hz ; 5Hz]. In order to build the elements of the coupled waveguide model from the analysis of real sounds, we are brought to deal with the inverse problem. Its solution leads to an estimate of the filters of the models at the resonance frequencies as function of the amplitudes, decay times and frequencies of the partials [4].

# 3. Results

Real signals have been collected from an experimental set up constituted of a massive support on which the two strings are tightened. The string velocity of each string is measured at a single location (near the bridge) by laser velocimetry. The velocity measured on the plucked string corresponds to the signal $S_1$ and the velocity measured on the string excited by sympathetic coupling corresponds to the signal $S_2$. As expected, the attack of the excited string is sharp whereas the amplitude of the signal from the coupled string increases slowly from zero. Figure 3 shows the whole spectrum obtained by Fourier transform of the signal, which contains more than fifty partials. Some of them seem to be missing: it is the phenomenon of rejection which depends on the location of both the excitation point and the measurement point.

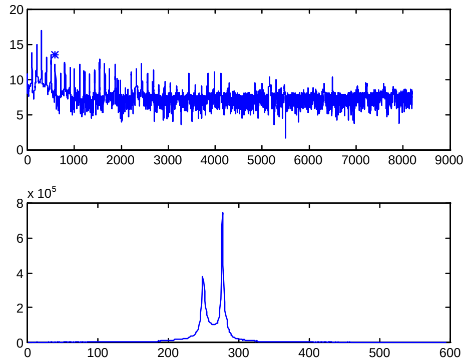


*Fig 3 : String excited by sympathetic coupling. The axes are arbitrary.*

*Top : spectrum of the real signal*

*Bottom: blow up of the measured spectrum around the fifth partial (marked by an asterisk on the top graph)*

Each partial is then isolated by band-pass filtering (fig 3, bottom). It shows a double resonance which introduces beats on the amplitude modulation law in the time domain. The two frequencies of each resonance correspond to the eigen frequencies of the coupled system which are different from the eigen frequencies of the uncoupled waveguide models. In a second step, a parametric method used on each doublet of partial in the time domain allows us to obtain temporal parameters (amplitudes, decay times and frequencies of the doublets), which are necessary to identify the values of the filters of the model at the resonance frequencies.

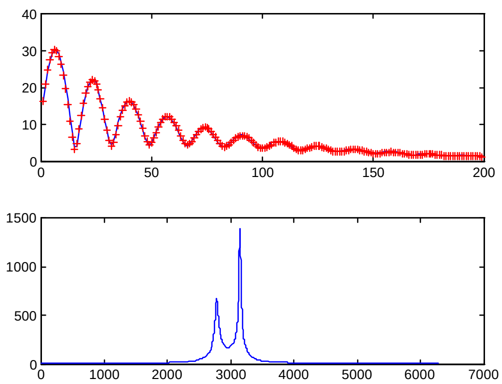

*Fig 4 : fifth partial, string excited by sympathetic coupling. The axes are arbitrary.*

*Top: superposition of the measured (-) and the estimated (+) amplitude modulation law*

*Bottom: blow up of the estimated spectrum*

By assuming that these functions are smooth enough in the frequency domain, they can be easily interpolated.

The modulus of the filters $F_1$ and $F_2$ are very close to one. Indeed, the propagation is weakly damped and this damping is due to the intrinsic losses within the string (viscoelastic, thermoelastic…) and to an energy transfer to the bridge [6]. The coupling filters (figure 5) show a peak for the third partial.

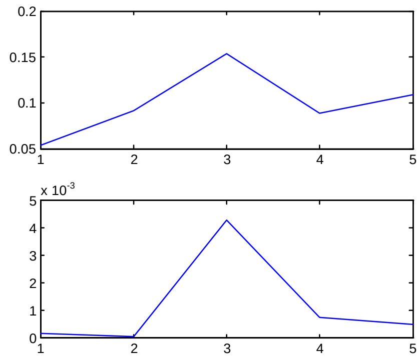


*Fig 5 : Modulus of the coupling filters F and H as functions of the index of the partials.*

The coupling is important at this frequency. In order to know if the behavior of the coupling filters is linked to the transfer function of the bridge, we have measured its response to a hammer shock while the strings were blocked. The analysis of the signal enabled us to obtain the frequency response of the bridge (figure 6).

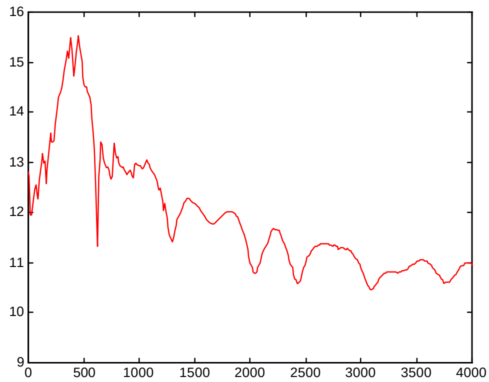

*Fig 6 : Frequency response of the bridge as function of the frequency (in Hz).*

It clearly shows a resonance around 470 Hz, which corresponds to the third partial of the coupled strings. The relation between the frequency response of the bridge and the values of the coupling filters thus seems obvious.

## 4. Synthesis with transformations

It is possible to achieve sound transformations by modifying some parameters of the model. First, there are two ways to transpose the sound, yielding different results:

- By changing the average frequencies of the partials before using them for building the filters. In this case, the transposition seems realistic as long as they are close to the initial frequencies; otherwise, the attack is not realistic.
- By modifying $D_1$ and $D_2$. This transformation is physically equivalent to change the length of the strings. The sound produced is satisfactory from a perceptive point of view.

We can also put the system out of tune by changing the phase of only one of the filters $F_1$ and $F_2$. In this case, the signal $S_2$ is much weaker than when strings are tuned and we can hear two distinct frequencies with beats. This result has a physical explanation: the fundamental frequencies of the two strings are distant, so the energy of the "first" string is not totally transferred to the "second" string. The partials, the frequencies of which are too different, are much weakly coupled. If the difference between the two frequencies is just one octave, the coupling becomes optimal and the string excited by coupling is vibrating. The intensity of the coupling is then related to the number of partials sharing the same frequency. The exchange between the filters $F_1$ and $F_2$ does not modify the features of the produced signal: those filters describe the internal loss phenomena in the two strings, which are supposed to be identical. The coupling is mainly associated to the filters F and H. Others parameters such as the excitation or the coefficient of inharmonicity (by acting on the phase of the two filters $F_1$ and $F_2$) can be changed to simulate other physical situations.

## 5. Conclusion

This study has shown that a coupled waveguides synthesis model is able to reproduce the perceptive effects of coupling between two piano strings, namely the phenomena of beats and double decay. By prolonging the work done in the case of a unique string with two polarisations to the case of two strings, we could determine the parameters of the model directly from the analysis of real sounds. For that purpose, we used a calibrated experimentation. From a mechanical point of view, we have seen that the behaviour of the coupling filters can be linked to the eigen modes of the bridge. From a musical point of view, the accuracy of the resynthetized sounds validates the use of this model and the techniques of parameters estimation. Though we worked on velocity signals from a specific experiment, the model in itself is valid in the case of real piano tones. Sound synthesis using this model gives the possibility, first of all, to transpose easily the sound and create ad infinitum new sounds by simply modifying the relevant parameters of the filters, which were estimated from the analysis. These new sounds can, for instance, correspond to simulations of systems of two strings with completely different physical properties (for example, one can virtually couple a nylon string of guitar and a steel string of piano).

## 6. References


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