Math 228
2-6-1994

Solutions to Problem Set 9.

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1. Find the solution of the differential equation $y'' + 2y' - 3y = 0$.

The characteristic equation is $m^2 + 2m - 3 = 0$. The roots are $m = -3$ and $m = 1$. The general solution is

$$y = c_1 e^{-3x} + c_2 e^x. \ \square$$

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5. Find the solution of the differential equation $y'' + 5y' = 0$.

The characteristic equation is $m^2 + 5m = 0$. The roots are $m = 0$ and $m = -5$. The general solution is

$$y = c_1 + c_2 e^{-5x}. \ \square$$

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6. Find the solution of the differential equation $4y'' - 9y = 0$.

The characteristic equation is $4m^2 - 9 = 0$. The roots are $m = \pm\frac{3}{2}$. The general solution is

$$y = c_1 e^{3x/2} + c_2 e^{-3x/2}. \ \square$$

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10. Find the solution of the initial value problem $y'' + 4y' + 3y = 0$, $y(0) = 2$, $y'(0) = -1$. Sketch the graph of the solution and describe its behavior as x increases.

The characteristic equation is $m^2 + 4m + 3 = 0$. The roots are $m = -1$ and $m = -3$. The general solution is

$$y = c_1 e^{-x} + c_2 e^{-3x}.$$

Find the derivative:

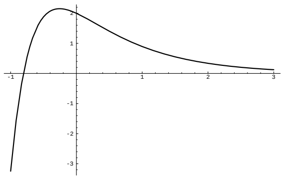
$$y' = -c_1 e^{-x} - 3c_2 e^{-3x}.$$

Plug in the initial conditions:

$$2 = y(0) = c_1 + c_2, \quad -1 = y'(0) = -c_1 - 3c_2.$$

Hence, $c_1 = \frac{5}{2}$ and $c_2 = -\frac{1}{2}$. The solution is

$$y = \frac{5}{2} e^{-x} - \frac{1}{2} e^{-3x}.$$



As $x \to \infty$, $y \to 0$. □

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1. Find the general solution of the differential equation $y'' - 2y' + y = 0$.

The characteristic equation is $m^2 - 2m + 1 = 0$. The root is $m = 1$ (double). The general solution is

$$y = c_1 e^x + c_2 x e^x. \ \square$$

8. Find the general solution of the differential equation $16y'' + 24y' + 9y = 0$.

The characteristic equation is $16m^2 + 24m + 9 = 0$. The root is $m = -\frac{3}{4}$ (double). The general solution is

$$y = c_1 e^{-3x/4} + c_2 x e^{-3x/4}. \ \square$$

14. Solve the initial value problem $y'' + 4y' + 4y = 0$, $y(-1) = 2$, $y'(-1) = 1$. Sketch the graph of the solution and describe its behavior for increasing x.

The characteristic equation is $m^2 + 4m + 4 = 0$. The root is $m = -2$ (double). The general solution is

$$y = c_1 e^{-2x} + c_2 x e^{-2x}.$$

Compute the derivative:

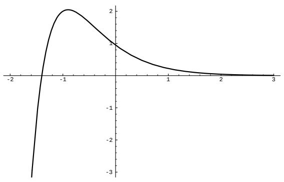
$$y' = (-2c_1 + c_2)e^{-2x} - 2c_2 x e^{-2x}.$$

Plug in the initial conditions:

$$2 = y(-1) = c_1 e^2 - c_2 e^2, \quad 1 = y'(-1) = (-2c_1 + c_2)e^2 + 2c_2 e^2.$$

Hence, $c_1 = 7e^{-2}$ and $c_2 = 5e^{-2}$. Therefore, the solution is

$$y = 7e^{-2}e^{-2x} + 5e^{-2}xe^{-2x} = (7 + 5x)e^{-2-2x}.$$



The negative exponential goes to 0 faster than the linear term goes to ∞. Hence, as $x \to \infty$, $y \to 0$. $\square$

"A crystal lacks rhythm from excess of pattern, while a fog is unrhythmic in that it exhibits a patternless confusion of detail." - Alfred North Whitehead

