Math 308
1-18-1994

Problem Set 1.

Problem Set 1 is due Wednesday, January 26.

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- In 4(d), use the fact that if I is maximal, then $\mathbb{Z}_n/I$ must be a field.
- In 24, first show A is an ideal. Then it suffices to show that $(\mathbb{Z} \oplus \mathbb{Z})/A$ is a field. It's a commutative ring with 1, so you need only show that a nonzero element $(a, b) + A$ is invertible. Consider the cases where $a = 1$ and $a = 2 \bmod 3$.
- In 25, first show A is an ideal. To show it's maximal, show R/A is a field. Take a nonzero element $f + A \in R/A$ and show it's invertible.
- In 33, you need $f(x)$ such that $(2x + 3)f(x) = 1 \bmod x^2 + x + 2$. Take $f(x) = ax + b$. Compute $(2x + 3)f(x) - 1$, divide it by $x^2 + x + 2$, and choose a and b so that the remainder is 0.

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- First, show that there is a multiplicative identity. Let $a \neq 0$. $aR = R$ and $a \in R$, so $ar = a$ for some $r \in R$. r looks like a candidate for 1. Prove that $rs = s$ for all $s \in R$. Finally, show that nonzero elements are invertible.

page 228: 13, 14, 26, 30

- In 14 (both parts), first write down the possible additive group maps. Such a map is determined by where 1 goes, and the order of $\phi(1)$ is limited by the orders of the groups. Then determine which of those satisfy $\phi(xy) = \phi(x)\phi(y)$.
- In 26, note that $(1, 0) = (1, 0)^2$. Apply ϕ to both sides. Now if $\phi(1, 0) = (a, b)$, what are the possibilities for a and b? Note that it isn't enough to say what is *possible*; you must show for each possibility (a, b) that there actually is a ring map which maps $(1, 0)$ to (a, b).
- In 30, look at the kernel of the map.

"To be idle requires a strong sense of personal identity." - *Robert Louis Stevenson*