Math 308
4-25-1994

Problem Set 10 (revised).

This problem set is due by Friday, April 29, at 3 p.m. No late papers will be accepted.

This is the revised version of Problem Set 10. I've changed the problems because we won't have time to discuss canonical forms or Smith reduction.

1. List all (isomorphism classes of) abelian groups of order 900. For each group in your list, give the invariant factor decomposition and the primary decomposition. *Be sure your list contains no repetitions!*

2. What is the minimal polynomial of

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}?$$

3. Let G be a finite abelian group whose order is not divisible by the square of a prime. Prove that G is cyclic.

4. Let $\mathbb{C}^*$ denote the multiplicative group of nonzero complex numbers. What is the torsion subgroup of $\mathbb{C}^*$?

5. Let H be the subset of $\mathbb{Z} \times \mathbb{Z}$ given by

$$H = \{(a, b) \mid a = b \bmod 4\}.$$

(a) Prove that H is a subgroup of $\mathbb{Z} \times \mathbb{Z}$.

(b) If H is not torsion free, find a nontrivial element of H having finite order. If H is torsion free, prove it, and show that H is isomorphic to a direct product of copies of $\mathbb{Z}$.