The Atomic Theory as Applied To Gases, with Some Experiments on the Viscosity of Air

by

Silas W. Holman

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

BACHELOR OF SCIENCE IN PHYSICS

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1876

© 1876 Silas W. Holman. All rights reserved.

The author hereby grants to MIT a nonexclusive, worldwide, irrevocable, royalty-free license to exercise any and all rights under copyright, including to reproduce, preserve, distribute and publicly display copies of the thesis, or release the thesis under an open-access license.

Authored by: Silas W. Holman
Department of Physics
May 18, 1876

Certified by: Edward C. Pickering
Professor of Physics, Thesis Supervisor

Accepted by: Tertius Castor
Professor of Log Dams
Graduate Officer, Department of Research

THESIS COMMITTEE

Thesis Supervisor

Edward C. Pickering
Professor of Physics
Department of Physics

Thesis Readers

Marcus Gavius Apicius
Professor of Cooking Arts
Department of Food Science

Marie-Antoine Carême
Professor of Haute Cuisine
Department of Food Science

Miles Gloriosus
Professor of Personal Pronouns
Department of Rhetoric

The Atomic Theory as Applied To Gases, with Some Experiments on the Viscosity of Air

by

Silas W. Holman

Submitted to the Department of Physics
on May 18, 1876 in partial fulfillment of the requirements for the degree of

BACHELOR OF SCIENCE IN PHYSICS

ABSTRACT

The developments of the "kinetic theory" of gases made within the last ten years have enabled it to account satisfactorily for many of the laws of gases. The mathematical deductions of Clausius, Maxwell and others, based upon the hypothesis of a gas composed of molecules acting upon each other at impact like perfectly elastic spheres, have furnished expressions for the laws of its elasticity, viscosity, conductivity for heat, diffusive power and other properties. For some of these laws we have experimental data of value in testing the validity of these deductions and assumptions. Next to the elasticity, perhaps the phenomena of the viscosity of gases are best adapted to investigation.[1]

Thesis supervisor: Edward C. Pickering
Title: Professor of Physics

[1] Text from Holman (1876): doi:10.2307/25138434.

Acknowledgments

Write your acknowledgments here.

Biographical Sketch

Silas Whitcomb Holman was born in Harvard, Massachusetts on January 20, 1856. He received his S.B. degree in Physics from MIT in 1876, and then joined the MIT Department of Physics as an Assistant. He became Instructor in Physics in 1880, Assistant Professor in 1882, Associate Professor in 1885, and Full Professor in 1893. Throughout this period, he struggled with increasingly severe rheumatoid arthritis. At length, he was defeated, becoming Professor Emeritus in 1897 and dying on April 1, 1900.

Holman's light burned brilliantly before his tragic and untimely death. He published extensively in thermal physics, and authored textbooks on precision measurement, fundamental mechanics, and other subjects. He established the original Heat Measurements Laboratory. Holman was a much admired teacher among both his students and his colleagues. The reports of his department and of the Institute itself refer to him frequently in the 1880's and 1890's, in tones that gradually shift from the greatest respect to the deepest sympathy.

Holman was a student of Professor Edward C. Pickering, then head of the Physics department. Holman himself became second in command of Physics, under Professor Charles R. Cross, some years later. Among Holman's students, several went on to distinguish themselves, including: the astronomer George E. Hale ('90) who organized the Yerkes and Mt. Wilson observatories and who designed the 200 inch telescope on Mt. Palomar; Charles G. Abbot ('94), also an astrophysicist and later Secretary of the Smithsonian Institution; and George K. Burgess ('96), later Director of the Bureau of Standards.

Contents

List of Figures 13

List of Tables 15

1 Introduction **17**

1.1 A section discussing the first issue: J/ψ 17

1.1.1 Subsection eqn. (1.2) 18

1.2 Description our paradigm 19

1.2.1 Conversion to a metaheuristic 20

1.3 Other generalizations 20

1.3.1 The most general case 20

1.4 Baroclinic generation of vorticity 21

1.5 Summary 21

Nomenclature for Chapter 1 22

A One-term coefficients for heat conduction **23**

A.1 A multipage table of numbers 23

References 25

List of Figures

1.1 A figure with two subfigures. 18
1.2 Caption text [1]. 20

List of Tables

1.1 The error function and complementary error function 22

A.1 One-term coefficients for one-dimensional heat conduction with a convective boundary condition. Data follow H. D. Baehr and K. Stephan [2]. 23

Chapter 1

Introduction

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

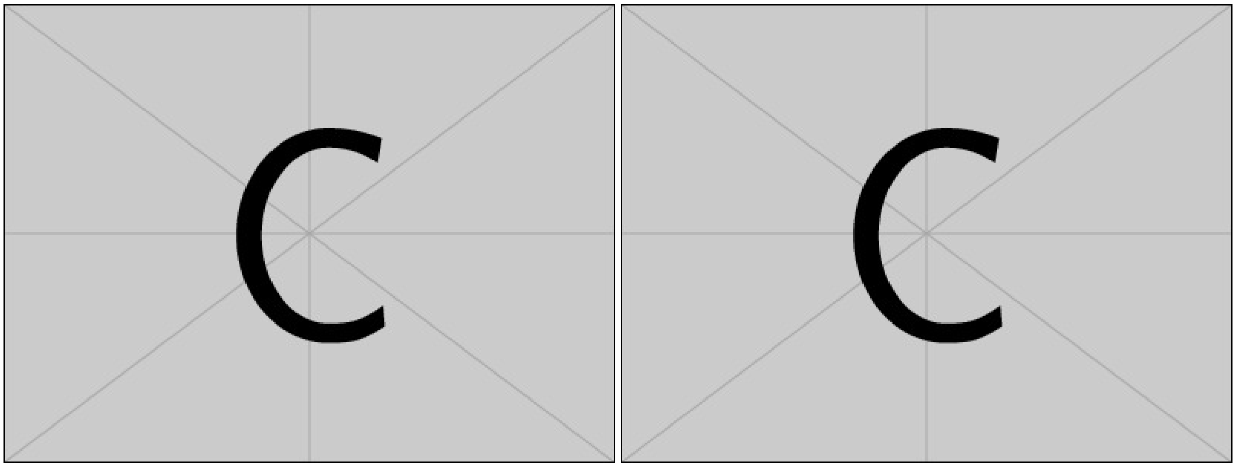
Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris. Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi, etsi sit elegans, personae tamen et dignitatis esse negent [3–7].

1.1 A section discussing the first issue: J/ψ

We begin with some ideas from the literature [8,9].

$$\frac{\partial}{\partial t}\left[\rho\left(e+|\vec{u}|^2/2\right)\right]+\nabla\cdot\left[\rho\left(h+|\vec{u}|^2/2\right)\vec{u}\right]=-\nabla\cdot\vec{q}+\rho\vec{u}\cdot\vec{g}+\frac{\partial}{\partial x_j}\left(d_{ji}u_i\right) \qquad (1.1)$$

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum



(a) Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi.

(b) Second subfigure.

Figure 1.1: A figure with two subfigures.

quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula. And more citations [1,9]. Then we write some more and include our citations [10–12]. The configuration is shown in Figs. 1(a) and 1(b).

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

1.1.1 Subsection eqn. (1.2)

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut

lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

A subsubsection

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

$$L(\mathbf{A}) = \begin{pmatrix} \frac{\varphi}{(\varphi_1,\varepsilon_1)} & 0 & \ldots & \ldots & \ldots & 0 \\ \frac{\varphi k_{2,1}}{(\varphi_2,\varepsilon_1)} & \frac{\varphi}{(\varphi_2,\varepsilon_2)} & 0 & \ldots & \ldots & 0 \\ \frac{\varphi k_{3,1}}{(\varphi_3,\varepsilon_1)} & \frac{\varphi k_{3,2}}{(\varphi_3,\varepsilon_2)} & \frac{\varphi}{(\varphi_3,\varepsilon_3)} & 0 & \ldots & 0 \\ \vdots & \vdots & & \ddots & & \vdots \\ \frac{\varphi k_{n-1,1}}{(\varphi_{n-1},\varepsilon_1)} & \frac{\varphi k_{n-1,2}}{(\varphi_{n-1},\varepsilon_2)} & \ldots & \frac{\varphi k_{n-1,n-2}}{(\varphi_{n-1},\varepsilon_{n-2})} & \frac{\varphi}{(\varphi_{n-1},\varepsilon_{n-1})} & 0 \\ \frac{\varphi k_{n,1}}{(\varphi_n,\varepsilon_1)} & \frac{\varphi k_{n,2}}{(\varphi_n,\varepsilon_2)} & \ldots & \ldots & \frac{\varphi k_{n,n-1}}{(\varphi_n,\varepsilon_{n-1})} & \frac{\varphi}{(\varphi_n,\varepsilon_n)} \end{pmatrix} \tag{1.2}$$

1.2 Description our paradigm

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui. No dissertation is complete without footnotes.[1,2,3]

[1] First footnote. $a_h = F_m$ See section 1.4.

[2] Another interesting detail.

[3] And another really important idea to have in mind [13–18].

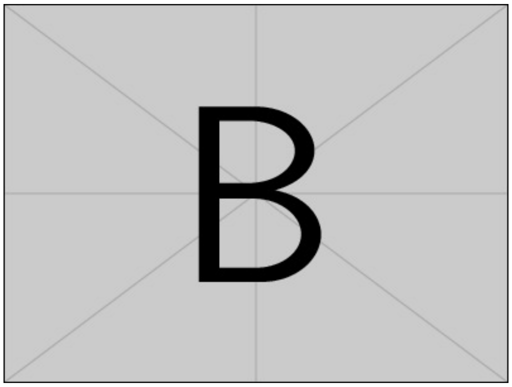


Figure 1.2: Caption text [1].

1.2.1 Conversion to a metaheuristic

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetuer tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo. This concept is discussed further in section 1.4, and Refs. [19,20].

1.3 Other generalizations

1.3.1 The most general case

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus. And another citation, so that our sources will

be unambiguous [21].

$$x\,\mathrm{Na(NH_4)HPO_4} \xrightarrow{\Delta} \mathrm{(NaPO_3)}_x + x\,\mathrm{NH_3}\uparrow + x\,\mathrm{H_2O} \tag{1.3}$$

$$^{234}_{90}\mathrm{Th} \longrightarrow {}^{0}_{-1}\beta + {}^{234}_{91}\mathrm{Pa} \tag{1.4}$$

$$\mathrm{SO_4}^{2-} + \mathrm{Ba}^{2+} \longrightarrow \mathrm{BaSO_4}\downarrow \tag{1.5}$$

$$\mathrm{Zn}^{2+} \underset{+2\,\mathrm{H}^+}{\overset{+2\,\mathrm{OH}^-}{\rightleftharpoons}} \underset{\text{amphoteric hydroxide}}{\mathrm{Zn(OH)_2}\downarrow} \underset{+2\,\mathrm{H}^+}{\overset{+2\,\mathrm{OH}^-}{\rightleftharpoons}} \underset{\text{tetrahydroxozincate}}{[\mathrm{Zn(OH)_4}]^{2-}} \tag{1.6}$$

These examples of chemical formulæ are copied directly from the documentation of the `mhchem` package, which was used to typeset them.

1.4 Baroclinic generation of vorticity

Substitution of the particle acceleration and application Stokes theorem leads to the *Kelvin-Bjerknes circulation theorem*, for $\rho \neq \mathrm{fn}(p)$:

$$\frac{d\Gamma}{dt} = \frac{d}{dt}\int_{\mathscr{C}} \mathbf{u}\cdot d\mathbf{r} \tag{1.7}$$

$$= \int_{\mathscr{C}} \frac{D\mathbf{u}}{Dt}\cdot d\mathbf{r} + \underbrace{\int_{\mathscr{C}} \mathbf{u}\cdot d\left(\frac{d\mathbf{r}}{dt}\right)}_{=0} \tag{1.8}$$

$$= \iint_{\mathscr{S}} \nabla\times\frac{D\mathbf{u}}{Dt}\cdot d\mathbf{A} \tag{1.9}$$

$$= \iint_{\mathscr{S}} \nabla p\times\nabla\left(\frac{1}{\rho}\right)\cdot d\mathbf{A} \tag{1.10}$$

Baroclinic generation of vorticity accounts for the sea breeze and various other atmospheric currents in which temperature, rather than pressure, creates density gradients. Further, this phenomenon accounts for ocean currents in straits joining more and less saline seas, with surface currents flowing from the fresher to the saltier water and with bottom current going oppositely.

1.5 Summary

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetuer tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in

Table 1.1: The error function and complementary error function

x	$\mathrm{erf}(x)$	$\mathrm{erfc}(x)$	x	$\mathrm{erf}(x)$	$\mathrm{erfc}(x)$
0.00	0.00000	1.00000	1.10	0.88021	0.11980
0.05	0.05637	0.94363	1.20	0.91031	0.08969
0.10	0.11246	0.88754	1.30	0.93401	0.06599
0.15	0.16800	0.83200	1.40	0.95229	0.04771
0.20	0.22270	0.77730	1.50	0.96611	0.03389
0.30	0.32863	0.67137	1.60	0.97635	0.02365
0.40	0.42839	0.57161	1.70	0.98379	0.01621
0.50	0.52050	0.47950	1.80	0.98909	0.01091
0.60	0.60386	0.39614	1.8214	0.99000	0.01000
0.70	0.67780	0.32220	1.90	0.99279	0.00721
0.80	0.74210	0.25790	2.00	0.99532	0.00468
0.90	0.79691	0.20309	2.50	0.99959	0.00041
1.00	0.84270	0.15730	3.00	0.99998	0.00002

cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl.

Nomenclature for Chapter 1

Roman letters

- $\mathscr{C}$ material curve
- $\mathbf{r}$ material position [m]
- $\mathbf{u}$ velocity [m s^{-1}]

Greek letters

- Γ circulation [m^2 s^{-1}]
- ρ mass density [kg m^{-3}]
- $\boldsymbol{\omega}$ vorticity [s^{-1}]

Appendix A

One-term coefficients for heat conduction

A.1 A multipage table of numbers

This example uses the `longtable` package: $\theta = A_1 f_1 \exp(-\lambda_1^2\,\mathrm{Fo})$, $\overline{\theta} = D_1 \exp(-\lambda_1^2\,\mathrm{Fo})$.

Table A.1: One-term coefficients for one-dimensional heat conduction with a convective boundary condition. Data follow H. D. Baehr and K. Stephan [2].

Bi	*Plate*			*Cylinder*			*Sphere*		
	λ_1	A_1	D_1	λ_1	A_1	D_1	λ_1	A_1	D_1
0.01	0.09983	1.0017	1.0000	0.14124	1.0025	1.0000	0.17303	1.0030	1.0000
0.02	0.14095	1.0033	1.0000	0.19950	1.0050	1.0000	0.24446	1.0060	1.0000
0.03	0.17234	1.0049	1.0000	0.24403	1.0075	1.0000	0.29910	1.0090	1.0000
0.04	0.19868	1.0066	1.0000	0.28143	1.0099	1.0000	0.34503	1.0120	1.0000
0.05	0.22176	1.0082	0.9999	0.31426	1.0124	0.9999	0.38537	1.0150	1.0000
0.06	0.24253	1.0098	0.9999	0.34383	1.0148	0.9999	0.42173	1.0179	0.9999
0.07	0.26153	1.0114	0.9999	0.37092	1.0173	0.9999	0.45506	1.0209	0.9999
0.08	0.27913	1.0130	0.9999	0.39603	1.0197	0.9999	0.48600	1.0239	0.9999
0.09	0.29557	1.0145	0.9998	0.41954	1.0222	0.9998	0.51497	1.0268	0.9999
0.10	0.31105	1.0161	0.9998	0.44168	1.0246	0.9998	0.54228	1.0298	0.9998
0.15	0.37788	1.0237	0.9995	0.53761	1.0365	0.9995	0.66086	1.0445	0.9996
0.20	0.43284	1.0311	0.9992	0.61697	1.0483	0.9992	0.75931	1.0592	0.9993
0.25	0.48009	1.0382	0.9988	0.68559	1.0598	0.9988	0.84473	1.0737	0.9990
0.30	0.52179	1.0450	0.9983	0.74646	1.0712	0.9983	0.92079	1.0880	0.9985
0.40	0.59324	1.0580	0.9971	0.85158	1.0931	0.9970	1.05279	1.1164	0.9974
0.50	0.65327	1.0701	0.9956	0.94077	1.1143	0.9954	1.16556	1.1441	0.9960
0.60	0.70507	1.0814	0.9940	1.01844	1.1345	0.9936	1.26440	1.1713	0.9944
0.70	0.75056	1.0918	0.9922	1.08725	1.1539	0.9916	1.35252	1.1978	0.9925
0.80	0.79103	1.1016	0.9903	1.14897	1.1724	0.9893	1.43203	1.2236	0.9904
0.90	0.82740	1.1107	0.9882	1.20484	1.1902	0.9869	1.50442	1.2488	0.9880
1.00	0.86033	1.1191	0.9861	1.25578	1.2071	0.9843	1.57080	1.2732	0.9855
1.10	0.89035	1.1270	0.9839	1.30251	1.2232	0.9815	1.63199	1.2970	0.9828
1.20	0.91785	1.1344	0.9817	1.34558	1.2387	0.9787	1.68868	1.3201	0.9800
1.30	0.94316	1.1412	0.9794	1.38543	1.2533	0.9757	1.74140	1.3424	0.9770
1.40	0.96655	1.1477	0.9771	1.42246	1.2673	0.9727	1.79058	1.3640	0.9739
1.50	0.98824	1.1537	0.9748	1.45695	1.2807	0.9696	1.83660	1.3850	0.9707

Table A.1: (continued)

Bi	Plate			Cylinder			Sphere		
	λ_1	A_1	D_1	λ_1	A_1	D_1	λ_1	A_1	D_1
1.60	1.00842	1.1593	0.9726	1.48917	1.2934	0.9665	1.87976	1.4052	0.9674
1.70	1.02725	1.1645	0.9703	1.51936	1.3055	0.9633	1.92035	1.4247	0.9640
1.80	1.04486	1.1695	0.9680	1.54769	1.3170	0.9601	1.95857	1.4436	0.9605
1.90	1.06136	1.1741	0.9658	1.57434	1.3279	0.9569	1.99465	1.4618	0.9570
2.00	1.07687	1.1785	0.9635	1.59945	1.3384	0.9537	2.02876	1.4793	0.9534
2.20	1.10524	1.1864	0.9592	1.64557	1.3578	0.9472	2.09166	1.5125	0.9462
2.40	1.13056	1.1934	0.9549	1.68691	1.3754	0.9408	2.14834	1.5433	0.9389
2.60	1.15330	1.1997	0.9509	1.72418	1.3914	0.9345	2.19967	1.5718	0.9316
2.80	1.17383	1.2052	0.9469	1.75794	1.4059	0.9284	2.24633	1.5982	0.9243
3.00	1.19246	1.2102	0.9431	1.78866	1.4191	0.9224	2.28893	1.6227	0.9171
3.50	1.23227	1.2206	0.9343	1.85449	1.4473	0.9081	2.38064	1.6761	0.8995
4.00	1.26459	1.2287	0.9264	1.90808	1.4698	0.8950	2.45564	1.7202	0.8830
4.50	1.29134	1.2351	0.9193	1.95248	1.4880	0.8830	2.51795	1.7567	0.8675
5.00	1.31384	1.2402	0.9130	1.98981	1.5029	0.8721	2.57043	1.7870	0.8533
6.00	1.34955	1.2479	0.9021	2.04901	1.5253	0.8532	2.65366	1.8338	0.8281
7.00	1.37662	1.2532	0.8932	2.09373	1.5411	0.8375	2.71646	1.8673	0.8069
8.00	1.39782	1.2570	0.8858	2.12864	1.5526	0.8244	2.76536	1.8920	0.7889
9.00	1.41487	1.2598	0.8796	2.15661	1.5611	0.8133	2.80443	1.9106	0.7737
10.00	1.42887	1.2620	0.8743	2.17950	1.5677	0.8039	2.83630	1.9249	0.7607
12.00	1.45050	1.2650	0.8658	2.21468	1.5769	0.7887	2.88509	1.9450	0.7397
14.00	1.46643	1.2669	0.8592	2.24044	1.5828	0.7770	2.92060	1.9581	0.7236
16.00	1.47864	1.2683	0.8541	2.26008	1.5869	0.7678	2.94756	1.9670	0.7109
18.00	1.48830	1.2692	0.8499	2.27556	1.5898	0.7603	2.96871	1.9734	0.7007
20.00	1.49613	1.2699	0.8464	2.28805	1.5919	0.7542	2.98572	1.9781	0.6922
25.00	1.51045	1.2710	0.8400	2.31080	1.5954	0.7427	3.01656	1.9856	0.6766
30.00	1.52017	1.2717	0.8355	2.32614	1.5973	0.7348	3.03724	1.9898	0.6658
35.00	1.52719	1.2721	0.8322	2.33719	1.5985	0.7290	3.05207	1.9924	0.6579
40.00	1.53250	1.2723	0.8296	2.34552	1.5993	0.7246	3.06321	1.9942	0.6519
50.00	1.54001	1.2727	0.8260	2.35724	1.6002	0.7183	3.07884	1.9962	0.6434
60.00	1.54505	1.2728	0.8235	2.36510	1.6007	0.7140	3.08928	1.9974	0.6376
80.00	1.55141	1.2730	0.8204	2.37496	1.6013	0.7085	3.10234	1.9985	0.6303
100.00	1.55525	1.2731	0.8185	2.38090	1.6015	0.7052	3.11019	1.9990	0.6259
200.00	1.56298	1.2732	0.8146	2.39283	1.6019	0.6985	3.12589	1.9998	0.6170
∞	1.57080	1.2732	0.8106	2.40483	1.6020	0.6917	3.14159	2.0000	0.6079

References

[1] M. Galassi, J. Davies, J. Theiler, B. Gough, G. Jungman, P. Alken, M. Booth, F. Rossi, and R. Ulerich. *GNU Scientific Library*. Version 2.4. Free Software Foundation. Boston, MA, 2017. URL: https://www.gnu.org/software/gsl/ (visited on 12/12/2018).

[2] H. D. Baehr and K. Stephan. *Heat and Mass Transfer*. Berlin: Springer-Verlag, 1998. ISBN: 3-540-63695-1.

[3] D. K. Edwards. “Radiative Transfer Characteristics of Materials.” *ASME J. Heat Transfer*, **91**(1), Feb. 1969, pp. 1–15. DOI: 10.1115/1.3580108.

[4] E. T. Whittaker and G. N. Watson. *A Course of Modern Analysis*. 3rd ed. Cambridge, UK: Cambridge University Press, 1920. URL: https://archive.org/details/courseofmodernan00whit.

[5] J. T. Kirk. *Decline and Fall of the Romulan Empire*. 7th ed. Humankind’s Greatest Writings 23. T’Paal: Vulcan Free Press, 2288. To appear.

[6] W. S. Churchill. *The Gathering Storm*. In: *The Second World War*. Vol. 1. Boston: Houghton Mifflin Co., 1948. Chap. 5, “The Locust Years”, pp. 66–89. URL: https://www.worldcat.org/oclc/3025315.

[7] J. W. Gibbs. “On the Form of the Teeth of Wheels in Spur Gearing.” Ph.D. dissertation. New Haven, CT: Yale University, 1863.

[8] C. Fong. *Analytical Methods for Squaring the Disc*. *ArXiv e-prints*. Sept. 2015. arXiv: 1509.06344.

[9] M. Sharpe. *New TX Font Package*. Version 1.71. Comprehensive TEX Archive Network, Mar. 2022. URL: https://ctan.org/pkg/newtx (visited on 05/29/2023).

[10] J. Swaminathan, R. L. Stover, E. W. Tow, D. M. Warsinger, and J. H. Lienhard. “Effect of Practical Losses on Optimal Design of Batch RO Systems.” In: *Proceedings of IDA World Congress on Desalination and Water Reuse* (São Paulo, Brazil, Oct. 15–20, 2017). IDA17WC-58334. International Desalination Association, Oct. 2017. HDL: 1721.1/111971.

[11] F. W. J. Olver, A. B. Olde Daalhuis, D. W. Lozier, B. I. Schneider, R. F. Boisvert, C. W. Clark, B. R. Miller, and B. V. Saunders, eds. *NIST Digital Library of Mathematical Functions*. Version 1.1.19. Gaithersburg, MD: National Institute of Standards and Technology, Mar. 2023. URL: https://dlmf.nist.gov/ (visited on 05/29/2023).

[12] American Mathematical Society and The LATEX Project. *User’s Guide for the amsmath Package*. Version 2.1. Comprehensive TEX Archive Network. Feb. 2020. URL: https://ctan.org/tex-archive/macros/latex/required/amsmath/amsldoc.pdf (visited on 05/29/2023).

[13] W. C. Reynolds, W. M. Kays, and S. J. Kline. *Heat Transfer in the Incompressible Turbulent Boundary Layer. I—Constant Wall Temperature*. NASA Technical Memorandum No. 12-1-58W. Washington, DC: National Aeronautics and Space Administration, Dec. 1958. HDL: 2060/19980228020.

[14] F. H. Clauser. “The Turbulent Boundary Layer.” In: *Advances in Applied Mechanics*. Ed. by H. L. Dryden and T. von Kármán. Vol. 4. Amsterdam, The Netherlands: Elsevier, 1956, pp. 1–51. DOI: 10.1016/S0065-2156(08)70370-3.

[15] J. H. Lienhard. “Heat Transfer in Flat-Plate Boundary Layers: A Correlation for Laminar, Transitional, and Turbulent Flow.” *ASME J. Heat Transfer*, **142**(6), 061805, June 2020. DOI: 10.1115/1.4046795.

[16] H. Johnson, ed. *Title of Edited Book*. New York, NY: John Wiley and Sons, Inc., 1980.

[17] H. W. Johnson, ed. *The Title of the Proceedings* (City, State, Apr. 27–28, 1965). Vol. 2. Organization. 1965. URL: https://news.mit.edu/2009/obit-johnson.

[18] S. van der Walt and N. Smith. *mpl Colormaps*. San Francisco, CA: GitHub, Sept. 2015. URL: https://bids.github.io/colormap/ (visited on 08/26/2018).

[19] L. Euler. “De Summis Serierum Reciprocarum.” *Commentarii Academiae Scientiarum Petropolitanae*, **7**, 1740, pp. 123–134. arXiv: math/0506415. First communicated to Daniel Bernoulli in 1734 and read before the St. Petersburg Academy in December 1735.

[20] J. B. J. Fourier. *Théorie Analytique de la Chaleur*. Paris: Firmin Didot, Père et Fils, 1822. URL: https://archive.org/details/analyticaltheory00fourrich.

[21] J. I. Montijano, M. Pérez, L. Rández, and J. L. Varona. “Numerical Methods With LuaLaTeX.” *TUGboat*, **35**(1), Jan. 2014, pp. 51–56. URL: https://tug.org/TUGboat/tb35-1/tb109montijano.pdf.