Math 228
3-20-1994

# Problem Set 22.

Problem Set 22 is due on Wednesday, March 23.

1. Compute $\mathcal{L}^{-1}\left(\dfrac{1}{(s-1)(s^2+1)}\right)$.

2. Solve for $y$ in terms of $t$:

$$y'' - 2y' + y = \begin{cases} t & \text{if } 0 \le t \le 1 \\ 2t+2 & \text{if } t > 1 \end{cases}, \qquad y(0) = 0, y'(0) = -1.$$

3. Solve for $y$ in terms of $t$:

$$y'' + 3y' + 2y = e^{-3t}u_2(t), \qquad y(0) = 2, y'(0) = -3.$$

4. Use Laplace transforms to compute $\displaystyle\int_0^\infty \frac{x\sin x}{x^2+1}\,dx$.

To do this, define

$$f(t) = \int_0^\infty \frac{x\sin(xt)}{x^2+1}\,dx.$$

The original integral is $f(1)$. Compute the Laplace transform $F(s)$ of $f(t)$. You may move the transform inside the integral, and then find $\mathcal{L}[\sin(xt)]$ using your transform table. This gives $F(s)$ as an integral with respect to $x$. Compute this integral using

$$\frac{x^2}{(x^2+1)(x^2+s^2)} = \frac{1}{1-s^2}\left(\frac{1}{x^2+1} - \frac{s^2}{x^2+s^2}\right).$$

This gives $F(s)$ in terms of $s$. Invert this to find $f(t)$, then plug in $t = 1$.

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**Reminders:** The **Review Session** is on Wednesday, March 23, at 7 p.m., in Bingham 103.

The **Test** is on Thursday, March 24, from 7 to 8:30 p.m., in Bingham 103.

All **homework** through Problem Set 22 must be turned in by 3 p.m., Monday, March 28.

The **computer project** on phase portraits is due on Friday, March 25.