Math 308
2-15-1994

Problem Set 4.

This is a revised version of Problem Set 4. It supersedes the original version that was handed out on Monday, February 14.

Problem Set 4 is due on Monday, February 21.

The first two problems should be done using *Mathematica.*
1. Write down your last name. Then let n be the number of letters in your last name. Substitute the value of n in the matrix below and write down the resulting matrix:

$$\begin{bmatrix} 3 & 0 & 7 & -1 & 1 & 4 \\ 2 & 3 & 2 & 0 & 0 & 6 \\ -2 & n & 3 & 1 & 1 & 1 \\ 2 & 3 & 1 & -9 & 0 & 4 \\ 0 & 2 & 0 & 3 & -3 & 5 \end{bmatrix}$$

Finally, use *Mathematica* to row reduce the matrix to row reduced echelon form.

Enter the matrix this way:

```
mat1 = {{3, 0, 7, -1, 1, 4},
    {2, 3, 2, 0, 0, 6},
    {-2, n, 3, 1, 1, 1},
    {2, 3, 1, -9, 0, 4},
    {0, 2, 0, 3, -3, 5}}
```

Obviously, you need to insert the appropriate value in place of `n`. To row reduce it, just do

```
RowReduce[mat1]
```

To see the row reduced matrix in standard matrix form, do:

```
RowReduce[mat1] // MatrixForm
```

2. Write down your last name. Then let n be the number of letters in your last name. Substitute the value of n in the matrix below and write down the resulting matrix:

$$\begin{bmatrix} 3 & 2 & -1 & 0 & 4 & 2 \\ 1 & 1 & n & 2 & -3 & 3 \\ -1 & -2 & 5 & 4 & 0 & -1 \\ 6 & 0 & -3 & 2 & 2 & 2 \end{bmatrix}$$

Then use *Mathematica* to find a basis for the null space of the matrix.

Enter the matrix as in Problem 1. Call it `mat2`. To find a basis for the null space, do:

```
NullSpace[mat2]
```

You may use *Mathematica* to do some of the problems below. If you use *Mathematica* to do a computation, be sure to say so.

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3. Let $V = \mathbb{R}^3$, considered as a vector space over $\mathbb{R}$.

(a) Let

$$\mathcal{S} = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}.$$

Find a basis $\mathcal{B}$ for $\mathbb{R}^3$ such that $\mathcal{S} \subset \mathcal{B}$. Prove that your set $\mathcal{B}$ is a basis.

(b) Let

$$\mathcal{T} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}.$$

Find a basis $\mathcal{B}$ for $\mathbb{R}^3$ such that $\mathcal{B} \subset \mathcal{T}$. Prove that your set $\mathcal{B}$ is a basis.

4. Let V be a vector space and let V_1 and V_2 be subspaces of V. The **sum** of the subspaces is

$$V_1 + V_2 = \{v_1 + v_2 \mid v_1 \in V_1, v_2 \in V_2\}.$$

Show that $V_1 + V_2$ is a subspace of V.

5. Let V be the vector space of functions $\mathbb{R} \to \mathbb{R}$.

(a) Let

$$V_e = \{f \in V \mid f(-x) = f(x) \text{ for all } x \in \mathbb{R}\},$$
$$V_o = \{f \in V \mid f(-x) = -f(x) \text{ for all } x \in \mathbb{R}\}.$$

(V_e is the set of **even functions** and V_o is the set of **odd functions**.) Show that V_e and V_o are subspaces of V.

(b) Show that $V_e + V_o = V$, but $V_e \cap V_o = \{0\}$. (The first statement means that you can write any function $\mathbb{R} \to \mathbb{R}$ as the sum of an even function and an odd function.)