Math 308
2-20-1994

Problem Set 5.

Problem Set 5 is due on Monday, February 28.

1. Row reduce the following matrix to row reduced echelon form *by hand.* You should say what operations you're performing at each step, particularly if you want to be lazy and perform several operations at a time. You may (and should) check your answer using *Mathematica*, but you won't get credit if you don't show your work.

$$\begin{bmatrix} 3 & 1 & -9 & -1 & 0 & 3 \\ 1 & 0 & -2 & 0 & 1 & 1 \\ -6 & -1 & 15 & 2 & -1 & -5 \end{bmatrix}$$

In the problems that follow, you may (and probably should) perform computations using *Mathematica*. However, if you use *Mathematica*, you should say exactly what commands you used — you'll have credit deducted if you simply write down the results without any explanation. (A printout of your *Mathematica* session is of course acceptable.)

2. Solve the following system of equations over $\mathbb{Z}_5$:

$$\begin{array}{ccccccccc} 3w & + & 2x & + & 4y & + & z & = & 1 \\ 2w & + & 3x & + & & & 2z & = & 1 \\ w & + & 3x & + & 4y & + & 2z & = & 3 \end{array}$$

You should write your solution in terms of the elements of $\mathbb{Z}_5$, i.e. $\{0,1,2,3,4\}$ — no fractions or negative numbers.

(You can row reduce as usual, then convert the answer to $\mathbb{Z}_5$. For example, if your answer contains $\dfrac{1}{7}$ before conversion, note that $7 = 2 \bmod 5$, and that $2^{-1} = 3 \bmod 5$.)

3. Replace n in the following matrix with the number of letters in your last name:

$$\begin{bmatrix} -2 & 4 & 5 & 0 & 2 & 1 \\ 1 & 3 & 0 & -4 & 6 & 7 \\ 1 & 2 & n & 2 & 4 & 5 \end{bmatrix}$$

Then find bases for the row space and the column space of the matrix.

4. Determine whether the vector $\langle -5,-16,10\rangle$ is in the subspace of $\mathbb{R}^3$ spanned by the vectors

$$\langle 1,-4,2\rangle, \langle 2,1,-1\rangle, \text{ and } \langle 4,11,-7\rangle.$$

You should either prove it isn't, or write it as a linear combination of the three vectors.

5. Let V be a vector space, and let V_1 and V_2 be subspaces of V. Suppose that $V = V_1 + V_2$, and $V_1 \cap V_2 = \{0\}$. Show that every vector $v \in V$ can be written *uniquely* as $v = x + y$, where $x \in V_1$ and $y \in V_2$.

6. Let

$$S = \left\{ \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 7 \end{bmatrix} \right\}.$$

Either find a subset of S which is a basis for $\mathbb{R}^4$, or show that no subset of S is a basis for $\mathbb{R}^4$.

7. Prove or disprove: If $\{x,y,z\}$ is an independent set of vectors in a real vector space V, then the set $\{x-y, y+z, x+2z\}$ is independent.

page 295: 2, 4, 8, 10, 13